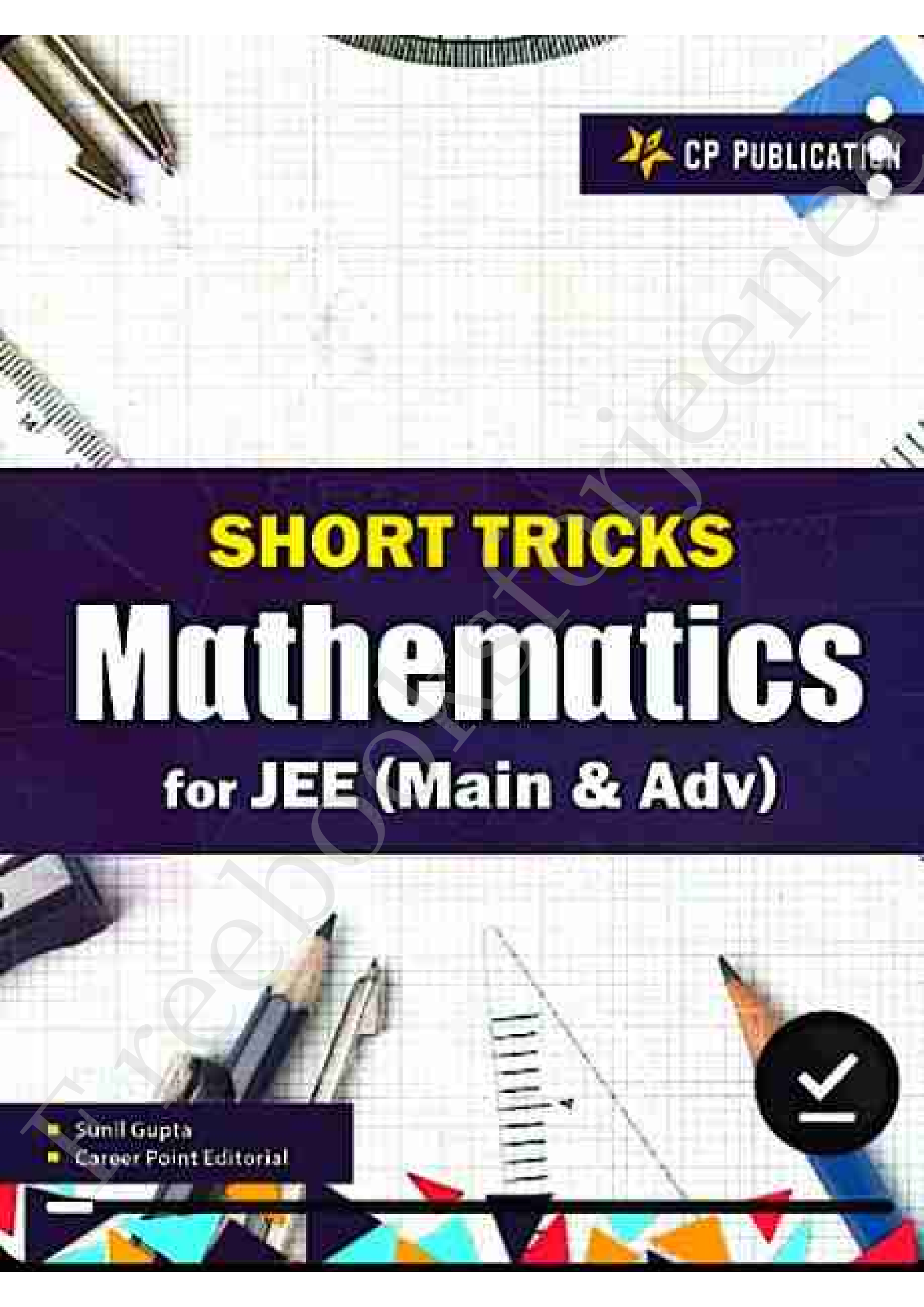
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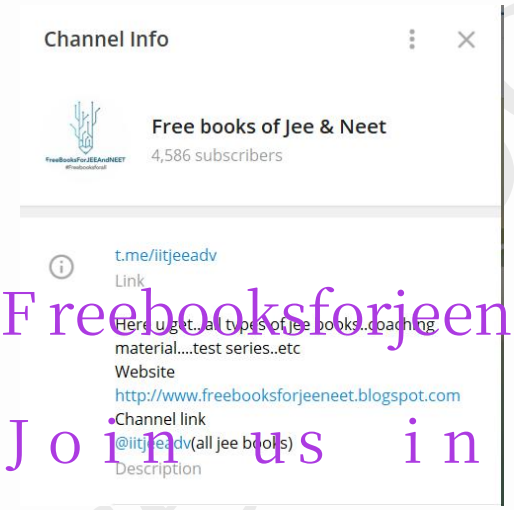

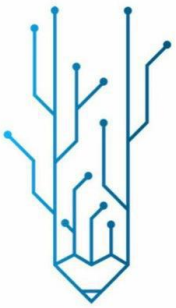

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4

Binomial Theorem

KEY CONCEPTS

1. Binomial Expression & Theorem

An algebraic expression containing two terms is called a **binomial expression.**
For example, $2x + 3$, $x^2 - x/3$, $x + a$ etc. are
The rule by which any power of a binomial can be expanded is called the **Binomial Theorem.**

2. Binomial Theorem for positive Integral Index

If x and a are two real numbers and n is a positive integer then
$(x + a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \ldots\ldots + {}^nC_r x^{n-r} a^r + \ldots\ldots + {}^nC_n x^0 a^n$.
Where ${}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, \ldots\ldots {}^nC_r \ldots\ldots$ are called **binomial coefficients** which can be denoted by $C_0, C_1, C_2, C_3 \ldots C_r$

3. General Term

In the expansion of $(x + a)^n$ the $(r + 1)^{th}$ term is called the **general term** which can be represented by T_{r+1}.
$T_{r+1} = {}^nC_r x^{n-r} a^r$

(i) General term in the expansion of $(x - a)^n$ is
$T_{r+1} = (-1)^r {}^nC_r x^{n-r} a^r$

(ii) General term in the expansion of $(1 + x)^n$ is
$T_{r+1} = {}^nC_r x^r$

(iii) General term in the expansion of $(1 - x)^n$ is
$T_{r+1} = (-1)^r {}^nC_r x^r$

4. Properties of Binomial Expansion $(x + a)^n$

(i) There are $(n + 1)$ terms in the expansion

(ii) The binomial coefficients of term equidistant from the beginning and the end are equal as ${}^nC_r = {}^nC_{n-r}$

(iii) The p^{th} term from the end of the expansion is the $(n - p + 2)^{th}$ term from the beginning.

5. Middle term in the Expansion of $(x + a)^n$

(i) If n is even then middle term = $\left(\frac{n}{2}+1\right)^{th}$ term.

(ii) If n is odd then middle terms are = $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ term. Binomial coefficient of middle term is the greatest Binomial coefficient.

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6. Greatest term in the Expansion of $(x + a)^n$

Calculate $\dfrac{n+1}{\left|\dfrac{x}{a}\right|+1} = k$ (say)

(i) If k is an integer then T_k and T_{k+1} are the numerically greatest terms.

(ii) If k is not an integer. Let m is its integral part then T_{m+1} is the numerically greatest term.

7. Binomial Coefficients & Their Properties

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_rx^r + \ldots + {}^nC_nx^n$.

Let us denote ${}^nC_0, {}^nC_1, {}^nC_2 \ldots {}^nC_n$ by $C_0, C_1, C_2 \ldots C_n$.

These quantities are called binomial coefficients and have the following properties.

(i) $C_0 + C_1 + C_2 + \ldots\ldots + C_n = \sum_{r=0}^{n} {}^nC_r = 2^n$

(ii) $C_0 - C_1 + C_2 - C_3 + \ldots (-1)^n C_n = \sum_{r=0}^{n} (-1)^r \, {}^nC_r = 0$

(iii) $C_0 + C_2 + C_4 + \ldots\ldots = 2^{n-1}$

(iv) $C_1 + C_3 + C_5 + \ldots\ldots = 2^{n-1}$

(v) $C_0 + 2C_1 + 3C_2 + \ldots + (n + 1)C_n = 2^{n-1}(n + 2)$

(ix) $C_0 + \dfrac{C_1}{2} + \dfrac{C_2}{3} + \ldots + \dfrac{C_n}{n+1} = \dfrac{2^{n+1} - 1}{n+1}$

(x) $C_0 - \dfrac{C_1}{2} + \dfrac{C_2}{3} - \dfrac{C_3}{4} + \ldots + \dfrac{(-1)^n . C_n}{n+1} = \dfrac{1}{(n+1)}$

(xi) $C_0C_r + C_1C_{r+1} + \ldots\ldots + C_{n-r}\,C_n = {}^{2n}C_{n-r}$

(xii) $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots\ldots + C_n^2 = {}^{2n}C_n$.

(xiii) $C_0C_1 + C_1C_2 + \ldots\ldots + C_{n-1}C_n = {}^{2n}C_{n-1}$

8. Binomial Theorem for any Index

When n is a negative integer or a fraction then the expansion of the binomial is possible only where

(i) Its first term is 1

(ii) Its second term is numerically less than 1

i.e. $|x| < 1$. Then

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots\ldots\ldots\infty$$

General term :

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{r!}x^r$$

9. Some Important Expansions

(i) $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \ldots\ldots + x^r + \ldots\ldots$ General term $T_{r+1} = x^r$

(ii) $(1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots\ldots (-x)^r + \ldots\ldots$ General term $T_{r+1} = (-x)^r$

(iii) $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots + (r + 1)x^r + \ldots\ldots$ General term $T_{r+1} = (r + 1)x^r$

(iv) $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots\ldots + (r + 1)(-x)^r + \ldots\ldots$ General term $T_{r+1} = (r + 1)(-x)^r$.

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10. R-f Factor Relation

If $(\sqrt{P} + Q)^n = I + f$ where I and n are the positive integers, and $0 \le f < 1$ then

(i) $(I + f)f = k^n$, If n is odd and $P - Q^2 = k > 0$

(ii) $(I + f)(1 - f) = k^n$, If n is even and $\sqrt{P} - Q < 1$

11. Multinomial Expansion

The number of distinct terms in the expansion of $(x_1 + x_2 + x_3 + ... + x_r)^m$ is given by ${}^{m+r-1}C_{r-1}$.

If $n \in N$ then the general terms of the multinomial expansion $(x_1 + x_2 + ...x_k)^n$ is

$$\frac{\lfloor n}{\lfloor a_1 \rfloor a_2 \rfloor a_3 .....\rfloor a_k} \cdot x_1^{a_1} . x_2^{a_2} .....x_k^{a_k}$$

where $a_1 + a_2 + a_3 + ...a_k = n$.

12. Number of terms in the Expansion of $(x + y + z)^n$

$= {}^{n+2}C_2$

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SHORT TRICKS

Trick -1 If the coefficient of T_r, T_{r+1}, T_{r+2} terms in the expansion of $(1+x)^n$ are in A.P., then the value of r is $r = \frac{n \pm \sqrt{n+2}}{2}$ $\forall\, n \in N$

Q.1 If the coefficients of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in A.P., then r -

(A) 6 (B) 7 (C) 8 (D) 9

Sol. **[D]**

Proper Method	**Short Trick**
Given ${}^nC_{r-1}$, ${}^nC_r$, ${}^nC_{r+1}$: AP $\because 2.{}^nC_r = nC_{r-1} + nC_{r+1}$ $\Rightarrow 2 = \frac{{}^nC_{r-1}}{{}^nC_r} + \frac{{}^nC_{r+1}}{{}^nC_r}$ $\Rightarrow 2 = \frac{1}{\left(\frac{n-r+1}{r}\right)} + \left(\frac{n-r-1+1}{r+1}\right)$ $\Rightarrow 2 = \frac{r}{n-r+1} + \frac{n-r}{r+1}$ Here n = 14 $\Rightarrow 2 = \frac{r}{15-r} + \frac{14-r}{r+1}$ $\Rightarrow 2(14r - r^2 + 15) = r^2 + r + 210 - 15r - 14r + r^2$ $\Rightarrow 4r^2 - 56r + 180 = 0 \Rightarrow r^2 - 14r + 45 = 0$ $\therefore r = 5, 9$	$r = \frac{14 \pm \sqrt{14+2}}{2} = 9$ or 5

Q.2 If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation - **[AIEEE-2005]**

(A) $m^2 - m(4r-1) + 4r^2 - 2 = 0$ (B) $m^2 - m(4r+1) + 4r^2 + 2 = 0$

(C) $m^2 - m(4r+1) + 4r^2 - 2 = 0$ (D) $m^2 - m(4r-1) + 4r^2 + 2 = 0$

Sol. **[C]**

Proper Method	**Short Trick**
Given ${}^mC_{r-1}$, ${}^mC_r$, ${}^mC_{r+1}$ are in AP $\Rightarrow 2.{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$ $\Rightarrow 2 = \left(\frac{{}^mC_{r-1}}{{}^mC_r}\right) + \left(\frac{{}^mC_{r-1}}{{}^mC_r}\right)$ $\Rightarrow 2 = \frac{1}{\left(\frac{m-r+1}{r}\right)} + \left(\frac{m-r-1+1}{r+1}\right)$ $\Rightarrow 2 = \frac{1}{\left(\frac{m-r+1}{r}\right)} + \left(\frac{m-r-1+1}{r+1}\right)$	$r = \frac{m \pm \sqrt{m+2}}{2}$ $\Rightarrow (2r-m)^2 = (m+2)$ $\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$

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$\Rightarrow 2 = \frac{r}{m-r+1} + \frac{m-r}{r+1}$ $\Rightarrow 2(m-r+1)(r+1) = r(r+1) + (m-r)(m-r+1)$ $\Rightarrow 2mr + 2m - 2r^2 - 2r + 2r + 2$ $= r^2 + r + m^2 - mr + m - mr + r^2 - r$ $\Rightarrow m^2 - 4mr - m + 4r^2 - 2 = 0$	

Q.3 If in the expansion of $(1+y)^n$, the coefficient of 5^{th}, 6^{th} and 7^{th} terms are in A.P., then n is equal to-

(A) 7, 11 (B) 7, 14 (C) 8, 16 (D) 7, 15

Sol. **[B]**

Proper Method	**Short Trick**
As given ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ are in AP. $\Rightarrow {}^nC_4 + {}^nC_6 = 2.\ {}^nC_5$ $\Rightarrow \frac{n!}{(n-4)!\,4!} + \frac{n!}{(n-6)!\,6!} = 2\,\frac{n!}{(n-5)!\,5!}$ $\Rightarrow 30 + (n-5)(n-4) = 2.6\,(n-4)$ $\Rightarrow n^2 - 21n + 98 = 0$ $\Rightarrow (n-7)(n-14) = 0$ $\therefore n = 7, 14$	$5 = \frac{n \pm \sqrt{n+2}}{2}$ $\Rightarrow (10-n)^2 = n + 2$ $\Rightarrow n^2 - 21n + 98 = 0$ $\Rightarrow n = 7, 14$

Trick -2 Method of substitution

Q.4 The fractional part of $\frac{2^{4n}}{15}$ is:

(A) $\frac{1}{15}$ (B) $\frac{2}{15}$ (C) $\frac{4}{15}$ (D) $\frac{7}{15}$

Sol. **[A]**

Proper Method	**Short Trick**
$\frac{2^{4n}}{15} = \frac{16^n}{15} = \frac{(1+15)^n}{15}$ $= \frac{1 + {}^nC_1 15 + {}^nC_2 15^2 + \ldots + {}^nC_n 15^n}{15}$ $= \frac{1+15k}{15}$, where $k \in N$ $= \frac{1}{15} + k$ $\therefore \left\{\frac{2^{4n}}{15}\right\} = \frac{1}{15}$	Put n = 1 $\Rightarrow$ fractional part of $\frac{16}{15}$ is $\frac{1}{15}$

Q.5 If the coefficients of four consecutive terms in the expansion of $(1+x)^n$ are a_1, a_2, a_3 and a_4, then $\frac{a_1}{a_1+a_2}, \frac{a_2}{a_2+a_3}, \frac{a_3}{a_3+a_4}$ are in -

(A) A.P. (B) G.P. (C) H.P. (D) None of these

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Sol. **[A]**

Proper Method	**Short Trick**
$a_1 = {}^nC_r$, $a_2 = {}^nC_{r+1}$, $a_3 = {}^nC_{r+2}$, $a_4 = {}^nC_{r+3}$ then $\frac{a_1}{a_1+a_2}, \frac{a_2}{a_2+a_3}, \frac{a_3}{a_3+a_4}$ $\Rightarrow \frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}, \frac{{}^nC_{r+1}}{{}^nC_{r+1}+{}^nC_{r+2}}, \frac{{}^nC_{r+2}}{{}^nC_{r+2}+{}^nC_{r+3}}$ $\Rightarrow \frac{{}^nC_r}{{}^{n+1}C_{r+1}}, \frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}}, \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}$ using ${}^nC_r = \frac{n}{r}.{}^{n-1}C_{r-1}$ we get $\Rightarrow \frac{1}{\left(\frac{n+1}{r+1}\right)}, \frac{1}{\left(\frac{n+1}{r+2}\right)}, \frac{1}{\left(\frac{n+1}{r+3}\right)}$ $\Rightarrow \frac{r+1}{n+1}, \frac{r+2}{n+1}, \frac{r+3}{n+1}$ $\Rightarrow (r+1), (r+2), (r+3)$: AP	Put n = 3 $\Rightarrow (1+x)^n = 1+3x+3x^2+x^3$ Here $a_1 = 1$, $a_2 = 3$, $a_3 = 3$, $a_4 = 1$ So, $\frac{a_1}{a_1+a_2} = \frac{1}{4}$, $\frac{a_2}{a_2+a_3} = \frac{3}{6}$, $\frac{a_3}{a_3+a_4} = \frac{3}{4}$ $\frac{1}{4}, \frac{3}{6}, \frac{3}{4}$ are in A.P.

Trick -3 Combination of method of substitution and balancing

Q.6 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ is equal to-

(A) $2^{n-1}(n+2)$ (B) $2^n(n+1)$ (C) $2^{n-1}(n+1)$ (D) $2^n(n+2)$

Sol. **[A]**

Proper Method	**Short Trick**
Putting x = 1 in the given expansion, we get $C_0 + C_1 + C_2 + C_3 + ... C_n = 2^n$ ...(1) Now, differentiating the given expansion with respect to x and then putting x =1, we get $C_1 + 2C_2 + 3C_3 + ... + nC_n = n.2^{n-1}$ Given Exp. $= C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ $= (C_0 + C_1 + C_2 + ... + C_n) + (C_1 + 2C_2 + 3C_3 + ... + nC_n)$ $= 2^n + n.2^{n-1}$ [from (1) and (2)] $= 2^{n-1}(n+2)$	Put n = 2 $\Rightarrow C_0 + 2.C_1 + 3.C_2$ $= 1 + 2(2) + 3(1) = 8$ Now put n = 2 in options then (A) $2^{n-1}(n+2)$ is correct option

Q.7 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $\frac{(C_0+C_1)(C_1+C_2)...(C_{n-1}+C_n)}{C_1C_2...C_n}$ equals-

(A) $\frac{n^n}{(n+1)!}$ (B) $\frac{(n+1)^n}{n!}$ (C) $\frac{n^n}{n!}$ (D) None of these

Sol. **[B]**

Proper Method	**Short Trick**
The given expression $= \frac{C_0C_1C_2...C_{n-1}}{C_1C_2...C_n} \cdot \left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right)...\left(1+\frac{C_n}{C_{n-1}}\right)$ $= \left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)\left(1+\frac{n-2}{3}\right)...\left(1+\frac{1}{n}\right)$ $(\because C_0 = C_n)$ $= \frac{(n+1)^n}{n!}$	Put n = 2 $\Rightarrow \frac{(C_0+C_1)(C_1+C_2)}{C_1C_2}$ $= \frac{(1+2)(2+1)}{(2)(1)} = \frac{9}{2}$ Now put n = 2 in option then (B) $\frac{(n+1)^n}{n!}$ is correct option.

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Q.8 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then $3C_0 - 5C_1 + 7C_2 + ...+ (-1)^n (2n+3) C_n$ equals-

(A) 1 (B) $2(2n + 3) 2^n$ (C) $(2n + 3) 2^{n-1}$ (D) 0

Sol. **[D]**

Proper Method	**Short Trick**
We have $3C_0 - 5C_1 + 7 C_2 + ...+ (-1)^n (2n+3) C_n$ $= 3C_0 - 3C_1 + 3C_2 + ...+ (-1)^n 3C_n - 2C_1 + 4C_2 + ...+ (-1)^n 2nC_n$ $= 3(C_0 - C_1 + C_2 + ...+ (-1)^n C_n) - 2(C_1 - 2C_2 + ... (-1)^n nC_n)$ $= 3 \times 0 - 2 \times 0 = 0.$	Put n = 2 $\Rightarrow 3C_0 - 5C_1 + 7C_2$ $= 3(1) - 5(2) + 7(1) = 0$ So, (D) is the correct option.

Q.9 If $(1+x)^n = 1 + C_1x + C_2x^2 + ...+ C_nx^n$, then $C_1 + C_3 + C_5 + ...$ is equal to-

(A) 2^n (B) $2^n - 1$ (C) $2^n + 1$ (D) 2^{n-1}

Sol. **[D]**

Proper Method	**Short Trick**
put $x = 1 \Rightarrow 2^n = 1 + C_1 + C_2 + C_3 ...$ put $x = -1 \Rightarrow 0 = 1 - C_1 + C_2 - C_3 ...$ subtracting, we get $2^n = 2 (C_1 + C_3 + C_5 ....)$ $\therefore C_1 + C_3 + C_5 ... = 2^{n-1}$	Put n = 3 $\Rightarrow C_1 + C_3 = 3 + 1 = 4$ Now put n = 3 in options, then (D) 2^{n-1} is correct option

Q.10 $n!\left(\frac{1}{n!}+\frac{1}{2!(n-2)!}+\frac{1}{4!(n-4)!}+.....+\frac{1}{n!}\right)$ is equal to -

(A) 2^n (B) 2^{n-1} (C) 2^{n+1} (D) 2^{-n+1}

Sol. **[B]**

Proper Method	**Short Trick**
$n!\left(\frac{1}{0!\,n!}+\frac{1}{2!\,n-2!}+\frac{1}{4!\,n-4!}+.....\right)$ $= \frac{n!}{0!\,n!} + \frac{n!}{2!\,n-2!} + \frac{n!}{4!\,n-4!} .....$ $= C_0 + C_2 + C_4 .....$ $= 2^{n-1}$	Put n = 2 $\Rightarrow 2!\left(\frac{1}{2!}+\frac{1}{2!\,(2-2)!}\right)$ $= 2\left(\frac{1}{2}+\frac{1}{2}\right) = 2$ Now put n = 2 in options then (B) 2^{n-1} is correct option

Q.11 $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ...=$

(A) $\frac{2^n}{n!}$ (B) $\frac{2^{n-1}}{n!}$ (C) 0 (D) $\frac{2^n}{n}$

Sol. **[B]**

Proper Method	**Short Trick**
$=\frac{1}{1!\,n-1!} + \frac{1}{3!\,n-3!} + \frac{1}{5!\,n-5!} + ...$ $= \frac{C_1}{n!} + \frac{C_3}{n!} + \frac{C_5}{n!} .....$ $= \frac{1}{n!} (C_1 + C_3 + C_5 .....)$ $= \frac{2^{n-1}}{n!}$	Put n = 3 $\Rightarrow \frac{1}{(1!)\,(2!)} + \frac{1}{(3!)(0!)}$ $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ Now put n = 3 in options then (B) $\frac{2^{n-1}}{n!}$ is correct option

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Q.12 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $C_0C_1 + C_1C_2 + C_2C_3 + ... + C_{n-1}C_n$ is equal to-

(A) $\frac{2n!}{n!\,n!}$ (B) $\frac{2n!}{n!(n+1)!}$ (C) $\frac{2n!}{(n-1)!\,(n+1)!}$ (D) $\frac{2n!}{(n-1)!\,n!}$

Sol. **[C]**

Proper Method

$C_0C_1 + C_1C_2 ..... + C_{n-1}C_n$

$= C_0C_{n-1} + C_1C_{n-2} ..... + C_{n-1}C_0$

= Coefficient of x^{n-1} in $(1+x)^n(1+x)^n$

= Coefficient of x^{n-1} is $(1+x)^{2n}$

$= {}^{2n}C_{n-1}$

Short Trick

Put n = 2

$\Rightarrow C_0C_1 + C_1C_2$

= (1) (2) + (2) (1) = 4

Now put n = 2 in options then (C) $\frac{(2n)!}{(n-1)!\,(n+1)!}$ is correct option

Q.13 ${}^nC_0 - \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 - .....+ (-1)^n \frac{{}^nC_n}{n+1} =$

(A) n (B) 1/n (C) $\frac{1}{n+1}$ (D) $\frac{1}{n-1}$

Sol. **[C]**

Proper Method

$\because (1+x)^n = C_0 + C_1x + C_2x^2 ..... + C_nx^n$

Integrating w.r.t. x from 0 to x

$$\int_0^x (1+x)^n dx = C_0\int_0^x dx + C_1\int_0^x x\,dx \ ... \ C_n\int_0^x x^n dx$$

$$\Rightarrow \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^x = C_0[x]_0^x + \frac{C_1}{2}[x^2]_0^x ... + \frac{C_n\left[x^{n+1}\right]_0^x}{n+1}$$

$$\Rightarrow \frac{(1+x)^{n+1}-1}{n+1} = C_0x + \frac{C_1x^2}{2} + ... + \frac{C_0x^{n+1}}{n+1}$$

put $\boxed{x=-1}$ in (i)

$$\Rightarrow \frac{-1}{n+1} = -C_0 + \frac{C_1}{2} - \frac{C_2}{3} + \frac{C_3}{4} ...$$

$$\therefore C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} ..... = \frac{1}{n+1}$$

Short Trick

Put n = 2

$\Rightarrow {}^2C_0 - \frac{1}{2}{}^2C_1 + \frac{1}{3}{}^2C_2$

$= 1 - \frac{2}{2} + \frac{1}{3} = \frac{1}{3}$

Now put n = 2 in options then (C) $\frac{1}{n+1}$ is correct option

Q.14 In the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$, the term independent of x is-

(A) $C_0^2 + 2C_1^2 + ...+ (n+1)C_n^2$ (B) $(C_0 + C_1 + ...+ C_n)^2$

(C) $C_0^2 + C_1^2 + ...+ C_n^2$ (D) None of these

Sol. **[C]**

Proper Method

$$(1+x)^n\left(1+\frac{1}{x}\right)^n$$

$$= (C_0 + C_1x + C_2x^2 ..... + C_nx^n)\left(C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} .... + \frac{C_n}{x^n}\right)$$

$\therefore$ Coefficient of independent term

$= C_0^2 + C_1^2 + C_2^2 ..... + C_n^2$

Short Trick

Put n = 1

$\Rightarrow (1+x)\left(1+\frac{1}{x}\right)$

$= 2 + x + \frac{1}{x}$ term in dependent of x is 2

Now put n = 1 in options then (C)

$(C_0^2 + C_1^2 + ... + C_n^2)$

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Q.15 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n$ is equal to-

(A) $\frac{2n!}{(n-r)!(n+r)!}$ (B) $\frac{2n!}{n!(n+r)!}$ (C) $\frac{2n!}{n!(n-r)!}$ (D) $\frac{2n!}{(n-1)!(n+1)!}$

Sol. **[A]**

Proper Method	**Short Trick**
$C_0C_r + C_1C_{r+1} + ...+ C_{n-r}C_n$ $= C_0C_{n-r} + C_1C_{n-r-1} .....+ C_{n-r}C_0$ = Coefficient of x^{n-r} in $(1+x)^n.(1+x)^n$ = Coefficient of x^{n-r} in $(1+x)^{2n}$ $= {}^{2n}C_{n-r}$	Put n = 2 and r = 2 $\Rightarrow C_0C_2 = (1)(1) = 1$ Now put n = 2 and r = 2 in options, then option (A) $\frac{(2n)!}{(n-r)!(n+r)!}$ is correct option

Q.16 If $C_0, C_1, C_2.....C_n$ are binomial coefficients in the expansion of $(1+x)^n$, $n \in N$, then $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + ...+ \frac{C_n}{n+1}$ is equal to -

(A) $\frac{2^{n+1}-1}{n+1}$ (B) $(n+1).2^{n+1}$ (C) $\frac{2^n-1}{n+1}$ (D) $\frac{2^n-1}{n}$

Sol. **[C]**

Proper Method	**Short Trick**
$\because (1+x)^n = C_0 + C_1x + C_2x^2 ..... + C_nx^n$ Integrating w.r.t. x from 0 to x $\int_0^x (1+x)^n dx = C_0\int_0^x dx + C_1\int_0^x x\,dx \ ... \ C_n\int_0^x x^n dx$ $\Rightarrow \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^x$ $= C_0[x]_0^x + \frac{C_1}{2}[x^2]_0^x ... + \frac{C_n[x^{n+1}]_0^x}{n+1}$ $\Rightarrow \frac{(1+x)^{n+1}-1}{n+1} = C_0x + \frac{C_1x^2}{2} + ... + \frac{C_0x^{n+1}}{n+1}$ ...(i) put $\boxed{x=-1}$ in (i) $\Rightarrow \frac{-1}{n+1} = -C_0 + \frac{C_1}{2} - \frac{C_2}{3} + \frac{C_3}{4} ...$ $\therefore C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} ..... = \frac{1}{n+1}$ ... (ii) Now put x = 1 in (i) $\Rightarrow \frac{2^{n+1}-1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4}$ .... (iii) Now subtracting (ii) from (iii), we get $\Rightarrow 2\left(\frac{C_1}{2} + \frac{C_3}{4} .....\right) = \frac{2^{n+1}-2}{n+1}$ $\therefore \frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} ..... = \frac{2^n-1}{n+1}$	Put $n = 3 \Rightarrow \frac{C_1}{2} + \frac{C_3}{4}$ $= \frac{3}{2} + \frac{1}{4} = \frac{7}{4}$ Now put n = 3 in options, then (C) $\frac{2^n-1}{n+1}$ is correct option

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Q.17 If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then for n odd, $C_1^2 + C_3^2 + C_5^2 + ...+ C_n^2$ is equal to

(A) 2^{2n-2} (B) 2^n (C) $\frac{(2n)!}{2(n!)^2}$ (D) $\frac{(2n)!}{(n!)^2}$

Sol. **[C]**

Proper Method	**Short Trick**
$\because C_0^2 + C_1^2 + C_2^2 \ldots + C_n^2 = C_0.C_0 + C_1.C_1 + \ldots + C_n.C_n$ $= C_0.C_n + C_1.C_{n-1} \ldots + C_nC_0$ = Coefficient of x^n in $(1 + x)^n .(1 + x)^n$ = Coefficient of x^n in $(1 + x)^{2n}$ $\therefore \boxed{C_0^2 + C_1^2 + C_2^2 + C_3^2 \ldots\ldots = {}^{2n}C_n}$ & $C_0^2 - C_1^2 + C_2^2 - C_3^2 \ldots\ldots$ $= C_0C_n - C_1C_{n-1} + C_2C_{n-2} - C_3C_{n-3} \ldots\ldots$ $= C_0.C_0 - C_1.C_1 + C_2.C_2 - C_3.C_3 \ldots\ldots$ = Coefficient of x^n in $(1 + x)^n (1 - x)^n$ = Coefficient of x^n in $(1 - x^2)^n$ n is odd so not possible $\therefore \boxed{C_0^2 - C_1^2 + C_2^2 - C_3^2 \ldots\ldots = 0}$ Subtracting (ii) from (i), we get $2(C_1^2 + C_3^2 + C_5^2 \ldots) = {}^{2n}C_n$ $\therefore C_1^2 + C_3^2 + C_5^2 \ldots = \frac{1}{2}.{}^{2n}C_n$	Put $n = 3 \Rightarrow C_1^2 + C_3^2$ $= 9 + 1 = 10$ Now put n = 3 in options then (C) $\frac{(2n)!}{2(n!)^2}$ is correct option

Q.18 If $(1+ x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then the value of $1^2 C_1 + 2^2 C_2 + 3^2 C_3 + ...+ n^2 C_n$ is

(A) $n(n + 1) 2^{n-2}$ (B) $n(n + 1) 2^{n-1}$ (C) $n(n + 1) 2^n$ (D) None of these

Sol. **[A]**

Proper Method	**Short Trick**
$\because C_0 + C_1x + C_2x^2 \ldots + C_nx^n = (1 + x)^n$ Differentiating w.r.t. x, we get - $\Rightarrow 1C_1 + 2C_2x \ldots\ldots + nC_nx^{n-1} = n(1 + x)^{n-1}$ Multiplying with w.r.t x, we get - $\Rightarrow 1C_1x + 2C_2x^2 \ldots\ldots + nC_nx^n = nx(1 + x)^{n-1}$ Differentiating w.r.t. x, we get - $\Rightarrow 1^2C_1 + 2^2C_2x \ldots\ldots + n^2C_nx^{n-1}$ $= n [(1 + x)^{n-1} + (n - 1)x (1 + x)^{n-2}]$ Put x = 1 $\Rightarrow 1^2C_1 + 2^2C_2 \ldots\ldots + n^2C_n = n(2^{n-1} + (n - 1).2^{n-2})$ $= n.2^{n-2} (2 + n - 1) = n (n + 1)2^{n-2}$	Put n = 2 $\Rightarrow 1^2 C_1 + 2^2 C_2$ $= (1) (2) + 4(1) = 6$ Now, put n = 2 in options, then option (A) $n(n + 1)2^{n-2}$ is correct option.

Q.19 The sum of the terms of the series $[3.{}^nC_0 - 8.{}^nC_1 + 13.{}^nC_2 - 18.{}^nC_3 + ...+ (n +1)]$ is $(n \in N, n \geq 2)$.

(A) $3.2^n - 5n.2^{n-1}$ (B) 0 (C) $3.2^n + 5n.2^{n-1}$ (D) $n.2^n$

Sol. **[B]**

Proper Method	**Short Trick**
$3C_0 - 8C_1 + 13C_2 - 18C_3 \ldots\ldots$ $= 3 [C_0 - C_1 + C_2 - C_3 \ldots] - 5[C_1 - 2C_2 + 3C_3 \ldots]$ $= 0 - 0 = 0$	Put n = 2 $\Rightarrow 3.{}^2C_0 - 8.{}^2C_1 + 13\ {}^2C_2$ $= 3(1) - 8(2) + 13(1) = 0$ Hence (B) is correct option

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Q.20 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_n.x^n$ then the value of $C_0C_n + C_1C_{n-1} + C_2C_{n-2} + ...+ C_nC_0$ is -

(A) 1 (B) $\frac{(2n)!}{(n)!\ (n)!}$ (C) $\left(\frac{(2n)!}{n!}\right)^2$ (D) $(2^n)^2$

Sol. **[B]**

Proper Method	**Short Trick**
$C_0C_n + C_1C_{n-1} ..... + C_nC_0$ = Coefficient of x^n in $(1+x)^n\ (1+x)^n$ = Coefficient of x^n in $(1+x)^{2n}$ $= {}^{2n}C_n$	Put n = 2 $\Rightarrow C_0C_2 + C_1C_1 + C_2C_0$ = (1) (1) + (2) (2) + (1) (1) = 6 Now put n = 2 in options, then option (B) $\frac{(2n)!}{(n)!\ (n)!}$ is correct option

Q.21 The middle term in the expansion of $\left(x+\frac{1}{2x}\right)^{2n}$ is-

(A) $\frac{1.3.5.....(2n-3)}{n!}$ (B) $\frac{1.3.5.....(2n-1)}{n!}$ (C) $\frac{1.3.5.....(2n+1)}{n!}$ (D) None of these

Sol. **[B]**

Proper Method	**Short Trick**
$\because$ 2n is even $\therefore \left(\frac{2n+2}{2}\right)^{th} = (n+1)^{th}$ is single middle term $\therefore T_{n+1} = {}^{2n}C_n.(x)^n\left(\frac{1}{2x}\right)^n$ $\therefore \boxed{T_{n+1} = \frac{{}^{2n}C_n}{2^n}}$ ... (i) But $^{2n}C_n = \frac{2n!}{n!.n!} = \frac{1.2.3.4....(2n-1).2n}{n!.n!}$ $= \frac{[1.3.5.....(2n-1)].(2.4.6.....2n)}{n!.\ n!}$ $= \frac{[1.3.5.....(2n-1)].2^n(1.2.3...n)}{n!\,(1.2.3....n)}$ $^{2n}C_n = \frac{1.3.5.....(2n-1)2^n}{n!}$ Putting in (i) : $T_{n+1} = \frac{1.3.5....(2n-1)}{n!}$	Put n = 1 $\Rightarrow \left(x+\frac{1}{2x}\right)^2$ $= x^2 + 1 + \frac{1}{4x^2}$ middle term is 1 Now put n = 1 in options, then (B) $\frac{1.3.5....(2n-1)}{(n)!}$ is correct option.

Q.22 If $(1+x)^n = \sum_{r=0}^{n} {}^nC_r.(x)^r$, then $\left(1+\frac{{}^nC_1}{{}^nC_0}\right).\left(1+\frac{{}^nC_2}{{}^nC_1}\right).....\left(1+\frac{{}^nC_n}{{}^nC_{n-1}}\right) =$

(A) $\frac{n^{n-1}}{(n-1)!}$ (B) $\frac{(n+1)^{n-1}}{(n-1)!}$ (C) $\frac{(n+1)^n}{n!}$ (D) $\frac{(n+1)^{n+1}}{n!}$

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Sol. **[C]**

Proper Method	**Short Trick**
$\left(1+\frac{n}{1}\right)\left(1+\frac{\frac{n(n-1)}{2}}{n}\right).....\left(1+\frac{1}{n}\right)$ $= (1+n)\frac{(1+n)}{2}\frac{(1+n)}{3}.....\frac{(1+n)}{n}$ $= \frac{(1+n)^n}{n!}$	Put $n = 2 \Rightarrow \left(1+\frac{^2C_1}{^2C_0}\right)\left(1+\frac{^2C_2}{^2C_1}\right)$ $= \left(1+\frac{2}{1}\right)\left(1+\frac{1}{2}\right) = \frac{9}{2}$ Now put n = 2 in options, then (C) $\frac{(n+1)^n}{(n)!}$ is correct option

Q.23 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + ... + \frac{nC_n}{C_{n-1}} =$ **[AIEEE-2002]**

(A) $\frac{n}{2}$ (B) $n(n+1)$ (C) $\frac{n(n+1)}{12}$ (D) $\frac{n(n+1)}{2}$

Sol. **[D]**

Proper Method	**Short Trick**
Let $S = \frac{1C_1}{C_0} + \frac{2C_2}{C_1} + ... + \frac{nC_n}{C_{n-1}}$ $\because T_r = r.\frac{C_r}{C_{r-1}} = r.\frac{n-r+1}{r}$ $\therefore T_r = (n+1) - r$ Now $S = \Sigma T_r = (n+1)\Sigma 1 - \Sigma r$ $= (n+1).n - \frac{(n+1)}{2}$ $\therefore S = \frac{n(n+1)}{2}$	Put n = 2 $\Rightarrow \frac{C_1}{C_0} + \frac{2.C_2}{C_1} = \frac{2}{1} + \frac{2(1)}{2} = 3$ Now, put n = 2 in options, then (D) $\frac{n(n+1)}{2}$ is correct option

Q.24 If $S_n = \sum_{r=0}^{n}\frac{1}{^nC_r}$ and $t_n = \sum_{r=0}^{n}\frac{r}{^nC_r}$, then $\frac{t_n}{s_n}$ is equal to- **[AIEEE-2004]**

(A) $\frac{1}{2}n$ (B) $\frac{1}{2}n - 1$ (C) $n - 1$ (D) $\frac{2n-1}{2}$

Sol. **[A]**

Proper Method	**Short Trick**
$S_n = \sum_{r=0}^{n}\frac{1}{^nC_r} = \frac{1}{^nC_0} + \frac{1}{^nC_1} ..... + \frac{1}{^nC_n}$ Now $t_n = \sum_{r=0}^{n}\frac{r}{^nC_r}$ $\therefore t_n = \frac{0}{^nC_0} + \frac{0}{^nC_1} ..... + \frac{n-1}{^nC_{n-1}} + \frac{n}{^nC_n}$ $\oplus\ t_n = \frac{0}{^nC_n} + \frac{1}{^nC_{n-1}} ..... + \frac{n-1}{^nC_{n-1}} + \frac{n}{^nC_0}$ $2t_n = n\left[\frac{1}{^nC_0} + \frac{1}{^nC_1} ..... + \frac{1}{^nC_n}\right] = nS_n$ $\therefore \frac{t_n}{S_n} = \frac{n}{2}$	Put n = 3 $\Rightarrow S_3 = \frac{1}{^3C_0} + \frac{1}{^3C_1} + \frac{1}{^3C_2} + \frac{1}{^3C_3}$ $= \frac{1}{1} + \frac{1}{3} + \frac{1}{3} + 1 = \frac{8}{3}$ $t_3 = \frac{0}{^3C_0} + \frac{1}{^3C_1} + \frac{2}{^3C_2} + \frac{3}{^3C_3}$ $= 0 + \frac{1}{3} + \frac{2}{3} + \frac{3}{1} = 4$ So, $\frac{t_n}{S_n} = \frac{3}{2}$ Put n = 3 in options then (A), $\frac{n}{2}$ is correct option

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Q.25 Sum of the series

$$2C_0 + \frac{C_1}{2}2^2 + \frac{C_2}{3}2^3 + \frac{C_3}{4}2^4 + \ldots\ldots \frac{C_n}{n+1}2^{n+1} \text{ equals-}$$

(A) $\frac{3^{n+1}-1}{n-1}$ (B) $\frac{3^{n+1}-1}{n+1}$ (C) $\frac{3^{n+1}+1}{n+1}$ (D) $\frac{3^{n-1}-1}{n+1}$

Sol. **[B]**

Proper Method

$$2C_0 + \frac{C_1}{2}2^2 + \frac{C_2}{3}2^3 + \ldots\ldots \frac{C_n}{n+1}2^{n+1}$$

$$= \sum_{r=0}^{n} \frac{1}{r+1} {}^nC_r\, 2^{r+1} = \frac{1}{n+1} \sum_{r=0}^{n} \frac{n+1}{r+1} {}^nC_r\, 2^{r+1}$$

$$= \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1}\, 2^{r+1} = \frac{1}{n+1} \sum_{s=1}^{n+1} {}^{n+1}C_s\, 2^s$$

$$= \frac{1}{n+1}\left[\sum_{s=0}^{n+1} {}^{n+1}C_s 2^s - {}^{n+1}C_0 2^0\right]$$

$$= \frac{1}{n+1}[(1+2)^{n+1}-1] = \frac{3^{n+1}-1}{n+1}$$

Short Trick

Put n = 2

$$\Rightarrow 2C_0 + \frac{C_1}{2}.2^2 + \frac{C_2}{3}.2^3$$

$$= 2(1) + \frac{2}{2}2^2 + \frac{1}{3}2^3$$

$$= 2 + 4 + \frac{8}{3} = \frac{26}{3}$$

Put n = 2 in options, then (B) $\frac{3^{n+1}-1}{n+1}$ is correct option.

Q.26 Coefficient of x^n in the expansion of $\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots\ldots\frac{x^n}{n!}\right)^2$ is-

(A) $\frac{2^{n+1}}{n!}$ (B) $\frac{2^{n-1}}{n!}$ (C) $\frac{2^n}{n!}$ (D) $\frac{2^n}{(n+1)}$

Sol. **[C]**

Proper Method

$$\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots\ldots\frac{x^n}{n!}\right)\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots\ldots\frac{x^n}{n!}\right)$$

$$\text{Coefficient of } x^n = 1.\frac{1}{n!} + \frac{1}{1!}.\frac{1}{(n-1)!} + \frac{1}{2!}\frac{1}{(n-2)!} + \ldots\frac{1}{n!}.1$$

$$= \frac{1}{n!}\left[\frac{n!}{0!\,n!} + \frac{n!}{1!(n-1)!} + \frac{n!}{2!(n-2)!} + \ldots\frac{n!}{n!\,0!}\right]$$

$$= \frac{1}{n!}[{}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots {}^nC_n]$$

$$= \frac{2^n}{n!}$$

Short Trick

Put n = 1 $\Rightarrow \left(1+\frac{x}{(1)!}\right)^2$

$= 1 + 2x + x^2$

Coeff. of x is 2

Now, put n = 1 in options, then, (C) $\frac{2^n}{(n)!}$ is correct option

Q.27 If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4}$ is equal to-

(A) $\frac{a_2}{a_2+a_3}$ (B) $\frac{1}{2}\frac{a_2}{a_2+a_3}$ (C) $\frac{2a_2}{a_2+a_3}$ (D) $\frac{2a_3}{a_2+a_3}$

Sol. **[C]**

Proper Method

Let $a_1 = {}^nC_r$

$\therefore \quad a_2 = {}^nC_{r+1}$

$a_3 = {}^nC_{r+2}$

$a_4 = {}^nC_{r+3}$

$$\therefore \frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4}$$

$$= \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} + \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}}$$

$$= \frac{{}^nC_r}{{}^{n+1}C_{r+1}} + \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}$$

$$= \frac{1}{\left(\frac{n+1}{r+1}\right)} + \frac{1}{\left(\frac{n+1}{r+3}\right)}$$

$$= \frac{1}{n+1}(2r+4) = \frac{2}{n+1}(r+2) = 2\left(\frac{r+2}{n+1}\right)$$

$$= 2.\ \frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}} = 2.\ \frac{a_2}{{}^nC_{r+1} + {}^nC_{r+2}}$$

$$= 2.\ \frac{a_2}{a_2+a_3}$$

Short Trick

Put n = 3

$\Rightarrow (1+x)^3 = 1 + 3x + 3x^2 + x^3$

Here $a_1 = 1$, $a_2 = 3$, $a_3 = 3$, $a_4 = 1$

So, $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4}$

$= \frac{1}{4} + \frac{3}{4} = 1$

After putting value of a_2 and a_3 (C) is correct option

Q.28 If $\frac{C_0}{1} - \frac{C_1}{3} + \frac{C_2}{5} - \ldots + \frac{(-1)^n C_n}{2n+1} = k\int_0^1 x(1-x^2)^{n-1}dx$, then k is :

(A) $\frac{2^{2n+1}n(n!)^2}{(2n+1)!}$ (B) $\frac{2^{2n+1}(n!)^2}{(2n+1)!}$ (C) $\frac{2^{n+1}n(n!)^2}{(2n+1)!}$ (D) None of these

Sol. **[A]**

Proper Method

$$\int_0^1 (1-x^2)^n dx = \int_0^1 (C_0 - C_1x^2 + C_2x^4 - \ldots)dx$$

$$= \frac{C_0}{1} - \frac{C_1}{3} + \frac{C_2}{5} - \ldots \qquad \ldots(1)$$

$$\therefore \frac{C_0}{1} - \frac{C_1}{3} + \frac{C_2}{5} - \ldots = k\int_0^1 x(1-x^2)^{n-1}dx \text{(say)}$$

$$= \frac{k}{2}\cdot\left[\frac{t^n}{n}\right]_0^1 = \frac{k}{2n}$$

Now $\int_0^1 (1-x^2)^n dx = \int_0^{\pi/2} \cos^{2n+1}\theta d\theta$

[$\sin\theta = x$]

Short Trick

Put n = 2

$$\Rightarrow \frac{C_0}{1} - \frac{C_1}{3} + \frac{C_2}{5} = k\int_0^1 x(1-x^2)dx$$

$$\Rightarrow 1 - \frac{2}{3} + \frac{1}{5} = k\left(\frac{x^2}{2} - \frac{x^4}{4}\right)_0^1$$

$$\Rightarrow k = \frac{+2}{15}$$

Put n = 2 in options, then (A) $\frac{2^{2n+1}n(n!)^2}{(2n+1)!}$ is correct option

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$$= \frac{\Gamma(n+1)\Gamma(1/2)}{2\Gamma\left(\frac{2n+3}{2}\right)}$$

$$\frac{n!\sqrt{\pi}}{2\left(\frac{2n+1}{2}\right)\left(\frac{2n-1}{2}\right)...(1/2)\sqrt{\pi}}$$

$$\Rightarrow \frac{k}{2n} = \frac{2^{2n}.n!n!}{(2n+1)!}$$

$$\Rightarrow k = \frac{2^{2n+1}.n(n!)^2}{(2n+1)!}$$

Q.29 If $C_r = {}^nC_r$ and $(C_0 + C_1)(C_1 + C_2) .... (C_{n-1} + C_n) = k\frac{(n+1)^n}{n!}$, then the value of k is:

(A) $C_0C_1C_2...C_n$ (B) $C_1^2C_2^2...C_n^2$ (C) $C_1 + C_2 + ... + C_n$ (D) None of these

Sol. **[A]**

Proper Method

$(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)...(C_{n-1} + C_n)$

$$= C_1C_2...C_n\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_1}{C_2}\right)...\left(1+\frac{C_{n-1}}{C_n}\right)$$

$$= C_1C_2...C_n\left(\frac{n+1}{n}\right)\left(\frac{n+1}{n}\right)..\left(\frac{1+n}{1}\right)$$

$$= C_1C_2...C_n, \frac{(n+1)^n}{n!}$$

$= C_0C_1C_2...C_n.(n+1)^n/n!$ $[C_0 = 1]$

$\therefore k = C_0C_1C_2...C_n$

Short Trick

Put n = 2

$\Rightarrow (C_0 + C_1)(C_1 + C_2)$

$$= k\frac{3^2}{(2)!}$$

$$\Rightarrow (1+2)(2+1) = k\frac{9}{(2)!}$$

$\Rightarrow k = 2$

Put n = 2 in options then (A) $C_0C_1C_2.....C_n$ is correct option

Q.30 If $x + y = 1$, then $\sum_{r=0}^{n} r^2\,{}^nC_r x^r y^{n-r}$ equals -

(A) nxy (B) nx (x + yn) (C) nx (nx + y) (D) None of these.

Sol. **[C]**

Proper Method

Given $(x + y) = 1$

$\because (x + y)^n = \Sigma {}^nC_r.x^{n-r}.y^r$

or $(x + y)^n = \Sigma {}^nC_r.x^r.y^{n-r}$

* Differentiating w.r.t. x, we get

$\Rightarrow n(x + y)^{n-1} = \Sigma r.{}^nC_r x^{r-1}.y^{n-r}$

* Multiplying x

$\Rightarrow nx(x + y)^{n-1} = \Sigma r.{}^nC_r.x^r.y^{n-r}$

*Differentiating w.r.t. x, we get

$\Rightarrow n[(x + y)^{n-1} + (n-1)x(x + y)^{n-2}] = \Sigma r^2.{}^nC_r.x^{r-1}.y^{n-r}$

Put $x + y = 1$

$\Rightarrow \Sigma r^2.{}^nC_r.x^{r-1}.y^{n-r} = n[1 + (n-1)x]$

$= n[1 + nx - x]$

$= n(nx + y)$

Multiplying by x, we get

$\Rightarrow \Sigma r^2.{}^nC_r.x^r.y^{n-r} = nx(nx + y)$

Short Trick

Put $x = \frac{1}{4}$, $y = \frac{3}{4}$, $n = 2$

$$= 0 + {}^2C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + 2^2.{}^2C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^0$$

Option (C) nx (nx + y) is correct option

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Q.31 Coefficients of x^r $[0 < r < (n-1)]$ in the expansion of

$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots\ldots + (x+2)^{n-1}$

(A) ${}^nC_r(3^r - 2^n)$ (B) ${}^nC_r(3^{n-r} - 2^{n-r})$ (C) ${}^nC_r(3^r + 2^{n-r})$ (D) ${}^nC_r(3^n - 2^n)$

Sol. **[B]**

Proper Method	**Short Trick**
Let $a = (x+3)$ & $b = (x+2)$ Now given series is : $a^{n-1} + a^{n-2}.b + a^{n-3}.b^2 \ldots\ldots + b^{n-1}$ $= a^{n-1}\left[1 + \frac{b}{a} + \left(\frac{b}{a}\right)^2 \ldots\ldots + \left(\frac{b}{a}\right)^{n-1}\right]$ $= a^{n-1}.1.\left[\frac{1-\left(\frac{b}{a}\right)^n}{1-\left(\frac{b}{a}\right)}\right] = \frac{a^n - b^n}{a-b}$ $= (x+3)^n - (x+2)^n = (3+x)^n - (2+x)^n$ $\therefore$ Coefficient of $x^r = {}^nC_r.3^{n-r} - {}^nC_r.2^{n-r}$	Put $r = 1$, $n = 3$ $= (x+3)^2 + (x+3)(x+2) + (x+2)^2$ $= 3x^2 + 15x + 19$ Coeff of $x = 15$ Now in option (B) ${}^nC_r(3^{n-r} - 2^{n-r})$ is correct option

Q.32 $1 - {}^nC_1\frac{1+x}{1+nx} + {}^nC_2\frac{1+2x}{(1+nx)^2} - {}^nC_3\frac{1+3x}{(1+nx)^3} + \ldots =$

(A) 0 (B) x^n (C) n^n (D) n

Sol. **[A]**

Proper Method	**Short Trick**
$1 - {}^nC_1\frac{1+x}{1+nx} + {}^nC_2\frac{1+2x}{(1+nx)^2} - {}^nC_3\frac{1+3x}{(1+nx)^3} + \ldots$ $= 1 - {}^nC_1\frac{1}{1+nx} + {}^nC_2\frac{1}{(1+nx)^2} - {}^nC_3\frac{1}{(1+nx)^3} + \ldots$ $-\frac{nx}{1+nx}\left\{1 - {}^{n-1}C_1\frac{1}{1+nx} + {}^{n-1}C_2\frac{1}{(1+nx)^2} - \ldots\right\}$ $= \left(1 - \frac{1}{1+nx}\right)^n - \frac{nx}{1+nx}\left(1 - \frac{1}{1+nx}\right)^{n-1}$ $= \frac{(nx)^n}{(1+nx)^n} - \frac{(nx)^n}{(1+nx)^n} = 0$	Put $n = 1 \Rightarrow 1 - \frac{1+x}{1+x} = 0$ By option (A) is correct option

Q.33 The sum of the series $\frac{2(n/2)!\,(n/2)!}{n!}\{C_0^2 - 2C_1^2 + 3C_2^2 - 4C_3^2 + \ldots + (-1)^n(n+1)C_n^2\}$ where n is an even positive integer, is equal to: **[IIT-1993]**

(A) $(-1)^{n/2}(n+1)$ (B) $(-1)^n(n+2)$ (C) $(-1)^{n/2}(n+2)$ (D) $(-1)^n(n-1)$

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Sol. **[C]**

Proper Method

$C_0 + C_1x + C_2x^2 + C_3x^3 + ...+C_nx^n = (1 + x)^n$ ..... (1)

Diff. (1) after multiplying by x, we get

$C_0 + 2C_1x + 3C_2x^2 + ... + (n + 1)C_nx^n$

$= (1 + x)^n + nx(1 + x)^{n-1}$ .... (2)

Also,

$C_0x^n - C_1x^{n-1} + C_2x^{n-2} - .... + x^n = (x - 1)^n = (1 - x)^n$

$\therefore\ C_0^2 - 2C_1^2 + 3C_2^2 - ... + (n+1)C_n^2$ [$\because$ n is even]

= coeff. of x^n in $(1 - x)^n \{(1 + x)^n + nx(1 + x)^{n-1}\}$

= coeff. of x^n in $\{(1 - x^2)^n + nx(1 - x)(1 - x^2)^{n-1}\}$

$= (-1)^{n/2}\,{}^nC_{n/2} - n.(-1)^{n-2/2}\,{}^{n-1}C_{(n-2)/2}$

$$= (-1)^{n/2}\left[\frac{n!}{(n/2)!(n/2)!} + \frac{n.(n-1)!(n/2)}{\{(n-2)/2\}!(n/2)!(n/2)}\right]$$

$$= (-1)^{n/2}.\frac{(n+2)}{2}.\frac{n!}{(n/2)!(n/2)!}$$

$$\therefore \frac{2(n/2)!(n/2)!}{n!}$$

$$\{C_0^2 - 2C_1^2 + 3C_2^2 - 4C_3^2 + ... + (-1)^n(n+1)C_n^2\}$$

$= (-1)^{n/2}(n + 2).$

Short Trick

Put n = 2

$$\Rightarrow \frac{2(1)!.(1)!}{(2)!}$$

$(C_0^2 - 2C_1^2 + 3C_2^2)$

$= (1 - 2(4) + 3) = -4$

Put n = 2 in option (C) $(-1)^{n/2}$ (n + 2) is correct option

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